

(Mis)Understanding the Atmospheric Neutrino Anomaly

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1. Introduction

Atmospheric neutrinos originate from the decay of unstable particles produced by cosmic ray interactions in the upper atmosphere. The useful spectrum runs from about 200 MeV to about 1330 MeV. But flux extends to much higher energies. The Super Kamioka group has studied these out to about 10 GeV. The higher energy sample is only about 21% of the size of the sample below 1330 MeV.

Atmospheric neutrinos approximate the ideal of a two distance neutrino oscillation experiment. Figure 1 shows the log of the flight length for neutrinos as a function of the cosine of the zenith angle. Events from below travel on the order of 8000 km. Those coming from above travel on the order of 10 km. There is rather little solid angle at intermediate distances. Atmospheric neutrinos provide a good mixture of electron and muon neutrinos since a significant number of the muons at these energies decay before reaching the ground. Simple arguments suggest that the ratio of muon to electron neutrinos should be about 2.

To summarize, atmospheric neutrinos are a mixture of muon and electron neutrinos coming from hadronic and muon decays. The source is approximately at two distances that differ by three orders of magnitude. Most of the flux spans about a factor of 7 in energy.

The atmospheric neutrino anomaly[1] is the observation of an apparent deficiency of muon type atmospheric neutrinos. The anomaly is believed to be evidence for neutrino oscillations[2]. But some of the details and assumptions used in the analysis cast some doubt on the oscillation hypothesis. The neutrino sample is being drawn from natural sources which adds a degree of un-

certainty.

Almost all analyses have assumed that the anomaly is restricted to the muon neutrinos and that the electron neutrino flux is a good normalizer.

2. $R(E_\nu)$

The ratio of observed to expected neutrino interactions is a measure of the possible reduction in the neutrino flux that is indicative of neutrino oscillations. Due to a significant uncertainty in the overall flux normalization, as much as 25%, the quality of the attenuation statistic is improved by normalizing to something which is better known. The value of $R = \frac{((\mu/e)_{Data})}{((\mu/e)_{MC})}$ [4] which attempts to normalize the muon neutrino flux to the electron neutrino flux is thought to have significantly smaller systematic errors, on the order of 5%.

An advantage that R has over other observables is that it maximizes the use of the data. It is insensitive to details of the angular correlation between neutrino direction and reconstructed lepton direction from the interaction. A disadvantage is that one is averaging over two different baselines so that if only the long distance sample has oscillated significantly the result will be diluted by the unoscillated sample from the shorter baseline.

2.1. The Fit to $R(E_\nu)$

Figure 2 illustrates the Super Kamioka measurement of R . The vertical scale is $\frac{1-R}{\sin^2(2\theta)}$ which combines the observed data, R with a fit parameter $\sin^2(2\theta)$. The horizontal axis of figure 2 is the \log_{10} of $\Delta m^2/E_\nu$ with Δm^2 in eV^2 and E_ν measured in eV. The points at the center of the plot are taken from references [5] and [6] and are

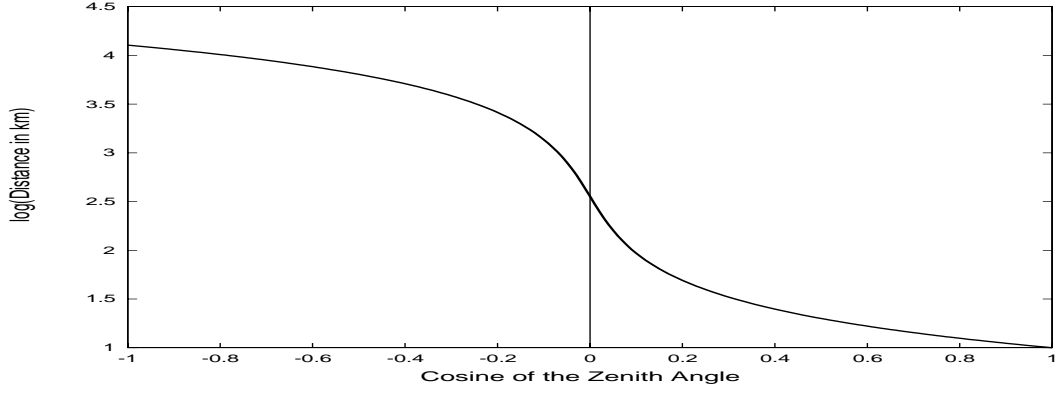


Figure 1. Log of the flight distance in km as a function of zenith angle. Downward going is on the right.

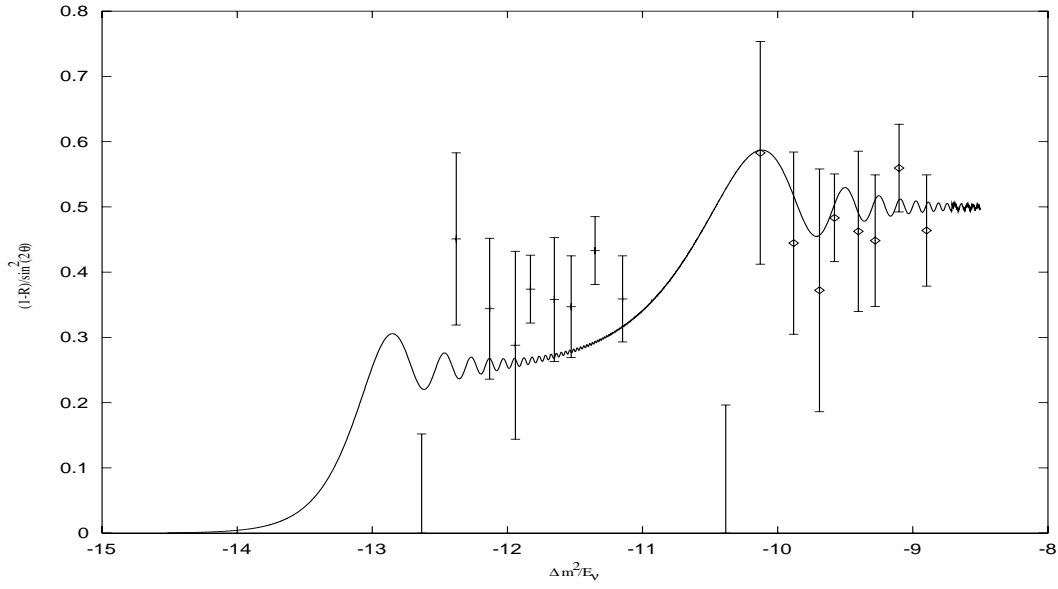


Figure 2. $\frac{1-R}{\sin^2(2\theta)}$ as a function of the \log_{10} of $\Delta m^2/E_\nu$ with Δm^2 in eV^2 and E_ν measured in eV. (Add 9 to get E_ν measured in GeV). The points plotted near the center of the figure are from references [5] and [6] using the Δm^2 from reference [7]. The points plotted to the right are our fit to Δm^2 using the data of [5] and [6].

plotted with $\sin^2(2\theta) = 1$ which is obtained from the Super Kamioka fit described in the next section. Since, for these point $\sin^2(2\theta) = 1$ the value of $1 - R$ can be read off the graph. Reference [5] quotes $R = 0.61 \pm 0.03 \pm 0.05$ for energies below 1330 MeV. Reference [6] quotes $R = 0.66 \pm 0.06 \pm 0.08$ for the higher energy sample.

The diamond points in figure 2 are the same data but the values of $\sin^2(2\theta)$ and Δm^2 are a fit to these data points. The fit yields $\sin^2(2\theta) = 0.8$ and $\Delta m^2 = 0.24 \text{ eV}^2$. The points near the center, the Super Kamioka fit, would agree with the data much better if the points could be lowered. But to do this would require $\sin^2(2\theta) = 1.5$ which is unphysical. It is difficult to assign a confidence level to our fit. The points share many common systematic errors. One would need to quantify these correlated errors in order to achieve a legitimate confidence interval. Since we have plotted $\Delta m^2/E_\nu$ the higher energy data points are to the left. The Δm^2 mass scale is driven by the low value of R measured for the higher energy data points. A reasonable fit can be obtained for $\Delta m^2 > 0.1 \text{ eV}^2$. More details of the fit can be found in [3].

The curve plotted in figure 2 is an integral over the distances of figure 1 for $1 - R$. For single component neutrino oscillations the result is a function of $\Delta m^2/E_\nu$ and proportional to $\sin^2(2\theta)$. The curve illustrates the structure expected from the two distance scales involved. At very small Δm^2 all path lengths are small compared to the oscillation length and so the value of R is 1. At very large Δm^2 the oscillation length is small compared to all path lengths in the problem so that $1 - R$ will be $0.5 \times \sin^2(2\theta)$, For a range of intermediate values of Δm^2 one path length is large compared to the oscillation length and the other is small so that $1 - R \approx 0.75 \times \sin^2(2\theta)$,

2.2. Geomagnetic Effects

Figure 3 illustrates the energy dependence of R for various values of Δm^2 . The figure was obtained by numerically integrating the Honda 97 muon neutrino flux[8]. For modest values of Δm^2 the value of R is energy independent over the range indicated. But it can never drop much

below 0.75. As Δm^2 rises portions of R drop to below 0.75 and start to approach 0.5. Only at values of Δm^2 of about 0.1 eV^2 is energy independence restored with $R < 0.75$. Figure 3 was generated for $\sin^2(2\theta) = 1$, its maximum value. For smaller values of $\sin^2(2\theta)$ the allowed values of R are greater. This plot supports the fit of figure 2 in that the only curve that gives an energy independent value for $R(E_\nu)$ at a value of R below ≈ 0.75 is $\Delta m^2 = 0.1 \text{ eV}^2$

The curve of figure 2 was done assuming an isotropic flux. A check using the Honda 97 flux reproduces the results to about 1%, except at the lowest energy point, $E=250 \text{ MeV}$, where the difference between isotropy and the calculated flux is less than about 3% for $\Delta m^2 \leq 1 \times 10^{-4} \text{ eV}^2$.

2.3. Physical Interpretation

The physical significance of a low value of R , if interpreted in terms of only a muon deficiency due to muon neutrino oscillations integrated over energy and distance, is that both distance scales must participate. Since the short distance scale is about 3 orders of magnitude shorter than the large one the favored value for Δm^2 must be comparably larger.

3. The Super Kamioka Fit

3.1. Systematic Errors

The Super Kamioka group has fit data to the neutrino oscillation hypothesis using:

$$\chi^2(\sin^2(2\theta), \Delta m^2, \vec{\epsilon}) = \sum_{i=1}^{70} \frac{(N_{data}^i - N_{MC}^i)^2}{\sigma_i^2} + \sum_j \frac{\epsilon_j^2}{\sigma_j^2}$$

σ_i includes the statistical error for the observed number of events in the i 'th bin. σ_j is the error on the j 'th fitted "systematic" parameter. σ_j is assumed to be known. $\vec{\epsilon}$ is a vector of 8 systematic parameters. The sum is over 70 bins; 5 bins in solid angle, 7 bins in energy for both electron and muon like events.

The number of Monte Carlo events is estimated from

$$N_{MC}^i = \frac{\mathcal{L}_{data}}{\mathcal{L}_{MC}} \times \sum_{MC \text{ events}} w$$

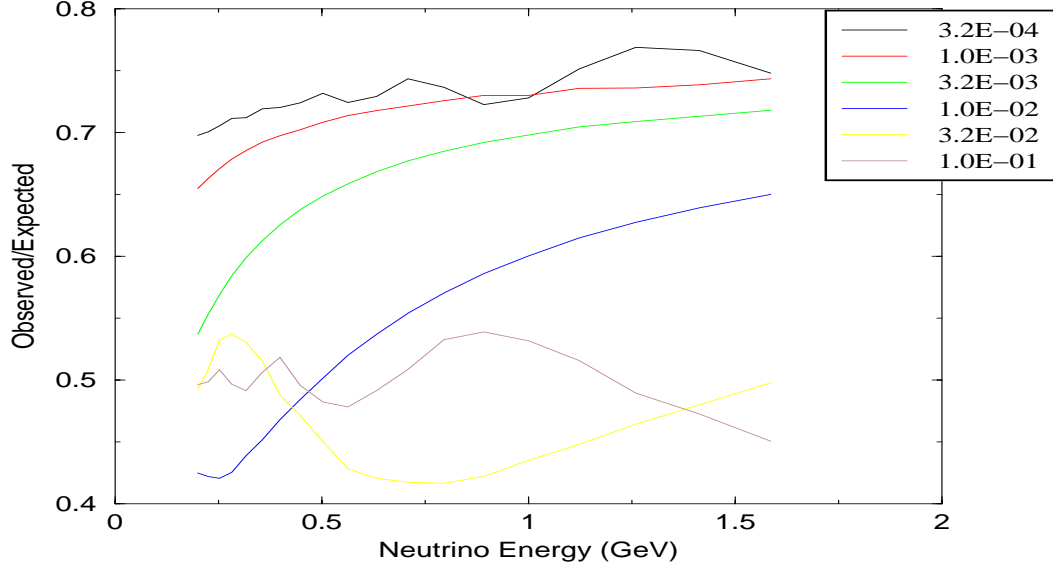


Figure 3. A plot of $R(E_\nu)$ for $\sin^2(2\theta) = 1$ and various Δm^2 . Δm^2 runs from $3.2 \times 10^{-4} \text{ eV}^2$ to 0.1 eV^2 as marked in the legend. These correspond in general to the ordering of the curves from top to bottom.

α	overall normalization
δ	E_ν spectral index
β_s	sub-GeV μ/e ratio
β_m	multi-GeV μ/e ratio
ρ	relative norm of PC to FC
λ	L/E_ν
η_s	sub-GeV up-down
η_m	multi-GeV up-down

Table 1

Super Kamioka fit parameters

Where the \mathcal{L} is the relative livetime of the data or Monte Carlo and

$$w = (1 + \alpha)(E_\nu^i/E_0)^\delta \left(1 + \frac{\eta_{s,m}}{2} \cos(\Theta)\right) \\ \times f_{e,\mu}(\sin^2(2\theta), \Delta m^2, (1 + \lambda)L/E_\nu) \\ \times \left\{ \begin{array}{l} (1 - \beta_s/2) \\ (1 + \beta_s/2) \\ (1 - \beta_m/2) \\ (1 + \beta_m/2)(1 - \frac{\rho}{2} \frac{N_{FC}}{N_\mu}) \\ (1 + \beta_m/2)(1 - \frac{\rho}{2}) \end{array} \right\}$$

The meaning of the various α , δ , η , β , λ and ρ are given in table 1. The choice of term from within the large braces depends on which portion of the data is being compared.

There are a number of problems with this fit. In particular the expression $(N_{data}^i - N_{MC}^i)^2$ includes a substantial systematic error in N_{MC}^i which is not included in the denominator σ_i^2 . Including this error would tend to *lower* the χ^2 .

But the systematic error in N_{MC}^i is common to many terms of the sum. In fact a definition of χ^2 including this common error would need to be of the matrix form:

$$\overline{(N_{data} - N_{MC})}^T (\sigma^2)^{-1} \overline{(N_{data} - N_{MC})}$$

Where $\overline{(N_{data} - N_{MC})}$ is a vector of all of the differences and $(\sigma^2)^{-1}$ is the weight matrix, the inverse of the covariance matrix, that includes the correlated errors.

Even with these definitions it would be hard to interpret the resulting χ^2 in terms of a confidence level since the *a priori* distributions of the systematic errors are not known.

3.2. The Asymptotic Assumption

The fit mentioned in the section above includes an unjustified assumption. It is assumed that the “systematic” parameters may be fit in an unbiased way. Many of these parameters are correlated with quantities of physical interest. For example the up to down flux ratio, η_s is closely tied to any observed angular asymmetry. The muon to electron neutrino flux ratio, β_s is essentially the denominator if the R value. The use of β_s and β_m effectively decouple the fit from the measured value of R .

The fit assumes that the systematic parameters may be fit to some portion of the solid angle where the data sample is not anomalous. In essence the fit assumes that the upper hemisphere represents a good sample of unoscillated events. But as discussed in section II the value of R makes this assumption rather risky. Any R interpreted solely in terms of muon neutrino oscillations must include the upper hemisphere. On the other hand, R could be anomalously low because the electron sample is not as expected.

The angular fit is relatively insensitive to a Δm^2 in the range of $5 \times 10^{-2} \text{ eV}^2$ to 10^{-4} eV^2 . This is because the data has been collected from primarily two baselines separated by 3 orders of magnitude. Δm^2 's over a rather broad range will give comparable fits to the data. It is only the relatively small amount of data near the horizon that is sensitive to these intermediate ranges of Δm^2 . In general it is easier to study such ranges by looking at the *energy* dependence after, perhaps, integrating over all solid angle, as we have done in the earlier section of this note.

4. Time Variation of the Event Rate and R

The event rate and the measured value of R for data collected during most of 1998[9] was significantly lower than that used in earlier Super Kamioka papers on atmospheric neutrinos[10]. In particular the overall event rate during the 321.6 live days of this period dropped by $12 \pm 3\%$ as compared to the event rate noted in the earlier 414.4 day period. A careful comparison of these newer data indicates that the muon event rate did not change. The 1998 muon rate was $3 \pm 5\%$ above the earlier rate. But both the electron sample and the multiprong event rate declined sharply. The electron sample was down by $18 \pm 5\%$. Table 2 summarizes the differences between the 1998 sample the earlier sample.

The change in the measured electron rate means that R must have changed. R rose from 0.61 ± 0.03 to 0.76 ± 0.04 . This rise in R brings it from a region which is not consistent with the hypothesis of neutrino oscillations over approximately half of the solid angle to a value consistent

	Cumulative[9]	Phys. Lett.[5]	1998 Sample
Single Ring	3224	1883	1341
e-like	1607	983	624
μ -like	1617	900	717
Multi-Ring	1271	784	487
Total	4495	2667	1828
μ/e	1.01 ± 0.04	0.92 ± 0.04	1.15 ± 0.06
$R = (\mu/e)_{Obs}/(\mu/e)_{MC}$	0.67 ± 0.02	0.61 ± 0.03	0.76 ± 0.04
Exposure (days)	736	414.4	321.6
Event Rate	6.11 ± 0.09	6.44 ± 0.12	5.68 ± 0.13
Single Ring Rate	4.38 ± 0.08	4.54 ± 0.10	4.17 ± 0.11
e rate	2.18 ± 0.05	2.37 ± 0.08	1.94 ± 0.08
μ rate	2.20 ± 0.05	2.17 ± 0.07	2.23 ± 0.08
Multi-Ring Rate	1.73 ± 0.05	1.89 ± 0.07	1.51 ± 0.07

Table 2

Comparison of Super Kamioka Sub-GeV Samples

with such a hypothesis. Unfortunately there is no good reason to doubt the credibility of either data sample. Super Kamioka has combined these samples in their analyses. The oscillation analysis utilizes the electron events, which are not consistent through this period.

R depends on the ratio of observations to simulations. The simulations did not change in any significant way between these two periods[10]

So there is clear evidence that the electron sample does not provide a reliable normalization for the analysis of atmospheric neutrino data. The assumption that the ratio of atmospheric electron to muon neutrino flux can be calculated to an accuracy of 5% is in error, unless some portion of the observed signal is not of atmospheric origin or is not neutrinos.[10]

The original Super Kamioka paper on the atmospheric neutrino flux[5] notes, “Using a detailed Monte Carlo simulation, the ratio $\frac{(\mu/e)_{Data}}{(\mu/e)_{MC}}$ was measured to be $0.61 \pm 0.03(stat) \pm 0.05(sys)$, consistent with previous results from the Kamiokande, IMB and Soudan-2 experiments, and smaller than expected from theoretical models of atmospheric neutrinos”. This is no longer true. The 1998 data is not consistent with previous results of the Super Kamioka detector or any of the earlier experiments cited. An R value of 0.76 ± 0.04 seems to be unique to this period.

The multi-GeV data sample has no evidence of

a rate variation in 1998.

Independent confirmation of the drop in event rate would help allay fears that it is due to instrumental or systematic effects. The Super Kamioka group has maintained very good monitoring of the detector performance [11] so such an explanation seems unlikely.

5. Conclusions

Several factors indicate that the atmospheric neutrino anomaly may be more complicated than generally thought. In such cases, it is important to seek corroboration of the results from other experiments. It is also important to be aware of simplifying assumptions that may have gone into the interpretation or the analysis of the data. One must be careful not to ignore information simply because it is in conflict with a preferred solution.

It would appear that the electron neutrinos can no longer be used for relative normalization, at least until the variation is understood. In the early days the anomaly was considered to be either a deficiency of muon like events or an excess of electron type events. In fact, the results looked a bit like a systematic error in the classification method used for electron and muon events. Since a random error would move events from the larger population, the muons, to the smaller population, the electrons. The observed event rate

for all events was much closer to most flux models than might be expected based the errors quoted for the estimates. Studies using the independent muon decay signature corroborate the morphology based results for muon and electron classification.

The “neutrino” events observed in underground detectors do not agree with expectations based on a simple source of atmospherically produced hadron and muon decays. Uncertainties in these expectations does not provide much guidance as to what might be wrong. Simple one component muon neutrino oscillations are unable to provide a convincing explanation for the small value of R nor for recent apparent drop in the interaction rate.

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